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Composite pulses for quantum computation with trapped electrons

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Abstract. In a scalable quantum computer, based on trapped electrons in vacuum, qubits are encoded in the external (cyclotron motion) and internal (spin) degrees of freedom. We show how to extend the technique of composite pulses to manipulate the cyclotron oscillator without leaving the computational subspace. In particular, we describe and discuss how to implement the explicit pulse sequence which operates the conditional phase shift between the cyclotron and the spin qubits.

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1 Introduction

Trapped particles, and especially ions in a Paul trap, are one of the most promising systems to implement a quantum computer. Since the seminal paper by Cirac and Zoller [1], many other theoretical proposals have dealt with trapped ions, accompanied as well by the first experimental tests of qubit manipulation [2–5]. Relevant progress has been made, but we are still at the proof-of-principle stage, with operations limited to very few qubits. Hence, it is then worthy to investigate in other directions, looking for alternative physical systems, and searching for different approaches to quantum computation.

In this exciting competition [6] also trapped electrons, stored in a so-called Penning trap [7], can play a significant role. The most obvious advantage offered by the electrons consists in the lighter mass (at least three orders of magnitude with respect to ions), which in turns results in higher trapping frequencies. In terms of quantum computation, this translates into a correspondingly faster clock frequency. Moreover, the Penning trap in comparison with RF ion traps promises weaker decoherence effects in view of the reduced fluctuations of its static electric and magnetic fields. Other relevant properties of a trapped electron are: (i) the ground state cooling and control of the cyclotron motion [8]; (ii) the extremely good isolation from the environment, leading to a negligible damping and reduced thermal fluctuations; (iii) the accurate preparation, manipulation, and detection of the electron state [9,10] that make the system suitable for high precision measurements and determination of fundamental constants, like the electron g factor [11] and mass [12].

Therefore, similarly to trapped ions, it has been proposed to store qubits into the internal (spin) and external (cyclotron and axial oscillators) degrees of freedom of a single electron in a Penning trap [13–15]. The fundamental issue of the scalability has then been faced in reference [16], suggesting an innovative design for a linear array of Penning traps [17]. Further extensions, towards a more compact and smaller device, are presently under investigation. They include, for example, the possibility of developing a planar Penning trap to form 2D trap arrays on the same substrate [18].

The present work has been motivated by the theoretical proposal by Childs and Chuang [19] to perform universal quantum computation with two-level systems. This is, indeed, the situation of the electron, when a qubit is encoded in the two possible orientations of its spin along the external magnetic field. In the case of ion traps, the other qubit is stored in the common center-of-mass motion, whereas in our case in the cyclotron oscillator. However, both systems are formally equivalent and face, therefore, the same problem of dealing with an harmonic oscillator storing one of the qubits. Being a multilevel system, a direct resonant excitation of the harmonic oscillator may lead to loose population outside the computational subspace. So far, for the electron in a Penning trap, it was suggested to use the rather small relativistic effects that introduce anharmonicities in the cyclotron oscillator [14,15]. The appealing feature of the technique developed in [19] is the possibility to avoid populating higher Fock states of the cyclotron motion, without perturbing the harmonic potential of the trap.

The manuscript is organized as follows. In Section 2, we introduce the theoretical model describing the quantized motion of a single electron in a Penning trap, with special emphasis on the cyclotron and spin degrees of

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freedom. The preparation and manipulation of the two qubits is the subject of Section 3, where we apply a multiple-pulse approach to construct the swapping gate and the conditional phase gate. Our results and concluding remarks are summarized in Section 4.

2 Electron in a Penning trap

The external dynamics of a single electron in a Penning trap consists of three independent motions: the magnetron, the axial, and the cyclotron oscillators [9]. Here, for simplicity, we restrict the analysis to the cyclotron oscillator plus the spin precession around the homogeneous static magnetic field of the trap. The cyclotron motion can be effectively described in terms of bosonic creation and annihilation operators

$$H_c = \hbar\omega_c \left(a_c^\dagger a_c + \frac{1}{2} \right), \tag{1}$$

since it is formally equivalent to a one-dimensional harmonic oscillator of frequency $\omega_c \simeq |e|B/m$, where B is the magnetic field strength, e and m are, respectively, the electron charge and mass. The energy eigenstates are given by the Fock states $|n\rangle$ with quantum number $n = 0, 1, 2, \ldots$ The corresponding eigenvalues build up a ladder of equally spaced energy levels, separated by $\hbar\omega_c$.

The electron spin dynamics is governed by the Hamiltonian

$$H_{spin} = -\boldsymbol{\mu} \cdot \mathbf{B} = \frac{g}{2}\mu_B \sigma_z B, \tag{2}$$

where g is the electron giromagnetic factor, $\mu_B \equiv |e|\hbar/2m$ is the Bohr magneton, and σ_z is a Pauli matrix. Hence, the two possible spin orientations along the magnetic field correspond to the states $|\uparrow\rangle$ and $|\downarrow\rangle$, separated by the energy $\hbar\omega_s$ with $\omega_s \equiv g|e|B/2m$. Note that the spin flip frequency ω_s slightly differs from the cyclotron frequency ω_c because of the electron anomaly.

Qubits are encoded in the cyclotron oscillator and spin states. The logical states 0 and 1 correspond, respectively, to $|\downarrow\rangle$ and $|\uparrow\rangle$. The combined cyclotron and spin states are conveniently denoted as $|pn\rangle$, with $p = 0, 1$ and $n = 0, 1, 2, \ldots$ The resulting energy level diagram is presented in Figure 1. The computational space is restricted to $\{|pn\rangle$ with $p, n = 0, 1\}$. However, being the cyclotron oscillator a multilevel system, it is not obvious how to confine the system dynamics to the lowest part of its spectrum. Indeed, a resonant excitation of the cyclotron motion would gradually populate Fock states with quantum number $n > 1$. A clever solution to circumvent this problem is provided by the so-called composite pulses technique [20], already successfully applied to trapped ions [5].

3 Qubit manipulation

The spin qubit is directly addressed via a small transverse magnetic field $\mathbf{b}(t)$ oscillating at the frequency ω, near to the spin resonance [14]

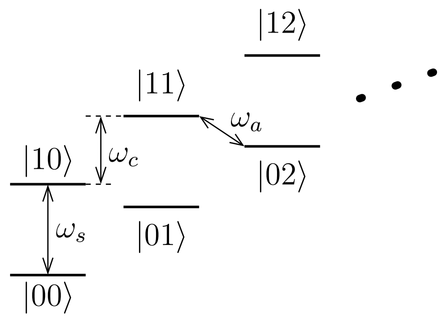

Fig. 1. Schematic diagram of the combined spin and cyclotron energy levels. The corresponding states are denoted by $|pn\rangle$, with $p = 0, 1$ being the logical value assigned to the spin orientation and $n = 0, 1, 2, \ldots$ the number of excitations in the cyclotron oscillator. We have also indicated the relevant frequencies of the system: the spin flip frequency ω_s, the cyclotron oscillation frequency ω_c, and the anomaly transition frequency $\omega_a \equiv \omega_s - \omega_c$.

$$H_{int} = -\boldsymbol{\mu} \cdot \mathbf{b}(t) = \frac{g}{2}\mu_B \boldsymbol{\sigma} \cdot \mathbf{b}(t), \tag{3}$$

where $\boldsymbol{\sigma} \equiv (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices and

$$\mathbf{b}(t) = b\left[\cos(\omega t)\mathbf{i} + \sin(\omega t)\mathbf{j}\right]. \tag{4}$$

The coupling between the spin magnetic moment and the applied magnetic field is proportional to the Rabi frequency

$$\Omega_s \equiv \frac{g\mu_B b}{\hbar} = \frac{g|e|b}{2m}. \tag{5}$$

Hence, the complete spin Hamiltonian results from equations (2) and (3)

$$H = H_{spin} + H_{int} = \frac{\hbar}{2}\omega_s\sigma_z + \frac{\hbar}{2}\Omega_s\left(\sigma_x \cos\omega t + \sigma_y \sin\omega t\right). \tag{6}$$

The effect of the oscillating magnetic field on the spin evolution becomes clearer if we move to a rotating frame at the frequency ω

$$H^{(IP)} = \exp\left(\frac{iH_0 t}{\hbar}\right) H \exp\left(-\frac{iH_0 t}{\hbar}\right), \tag{7}$$

with $H_0 = \hbar\omega\sigma_z/2$. The spin dynamics is then governed by the Hamiltonian in interaction picture (IP)

$$H^{(IP)} = \frac{\hbar}{2}(\omega_s - \omega)\sigma_z + \frac{\hbar}{2}\Omega_s\sigma_x. \tag{8}$$

Therefore, in the new rotating frame the electron spin precesses at the frequency

$$\Omega \equiv \sqrt{\Omega_s^2 + (\omega_s - \omega)^2} \tag{9}$$

around the axis

$$\mathbf{n} = \frac{\Omega_s}{\Omega}\mathbf{i} + \frac{\omega_s - \omega}{\Omega}\mathbf{k}. \tag{10}$$

When the external magnetic field is tuned on resonance with the spin flip transition frequency ω_s, the rotation axis is directed along the x-axis. According to the interaction time, an initially up (down) spin can be flipped into a down (up) state. In terms of qubit state manipulation, by changing the interaction time one can select any rotation angle, thus preparing the electron spin in a linear superposition of the logical states $|0\rangle$ and $|1\rangle$ [14]

$$|0\rangle = |0\rangle \cos\left(\frac{\Omega_s}{2}t\right) - i|1\rangle \sin\left(\frac{\Omega_s}{2}t\right), \tag{11}$$

$$|1\rangle = |1\rangle \cos\left(\frac{\Omega_s}{2}t\right) - i|0\rangle \sin\left(\frac{\Omega_s}{2}t\right). \tag{12}$$

A similar procedure cannot be applied to the cyclotron qubit because we are dealing with a multilevel system of equally spaced energy levels. Starting from the ground state of the cyclotron oscillator, the interaction with a resonant field would soon spread the population outside the computational subspace. Working, instead, at the anomaly frequency $\omega_a \equiv \omega_s - \omega_c$ one would only populate the extra level $|02\rangle$, as it can be seen from Figure 1. Actually, the spin-cyclotron level $|02\rangle$ is later depleted by means of another pulse of suitable duration.

From the practical point of view, the anomaly transition, which corresponds to a spin flip associated to a quantum jump of the cyclotron oscillator, is produced with a small magnetic field near the trap center. In previous experiments for the determination of the electron anomaly, the upper and lower portions of the trap electrodes were split in order to obtain two effective current loops [9]. The loops are then driven with oppositely directed currents at the frequency ω_d

$$I(t) = I\cos(\omega_d t + \phi), \tag{13}$$

where I and ϕ represent, respectively, the intensity and the phase of the alternating currents. This set-up produces the required magnetic field

$$\mathbf{b}_1(t) = b_1\left[x(t)\mathbf{i} + y(t)\mathbf{j}\right]\cos(\omega_d t + \phi), \tag{14}$$

with amplitude

$$b_1 \equiv \frac{3\pi a^2 dI}{c(a^2 + d^2/4)^{5/2}}, \tag{15}$$

depending on the loop radius a and distance d. The electron spin is driven by the oscillatory field $\mathbf{b}_1(t)$ according to the Hamiltonian

$$\begin{aligned} H_{drive} &= -\boldsymbol{\mu}\cdot\mathbf{b}_1(t) \\ &= \frac{g}{2}\mu_B b_1\left[\sigma_x x(t) + \sigma_y y(t)\right]\cos(\omega_d t + \phi). \end{aligned} \tag{16}$$

The alternating magnetic field introduces a coupling between the circular motion of the electron in the xy-plane and its spin. The interaction Hamiltonian, equation (16), can be recasted in terms of the raising and lowering operators for the spin motion

$$\sigma_\pm \equiv \frac{\sigma_x \pm i\sigma_y}{2}, \tag{17}$$

and of the annihilation and creation operators for the cyclotron and magnetron oscillators [14]

$$x \equiv \sqrt{\frac{\hbar}{2m\tilde{\omega}_c}}\left(a_c + a_c^\dagger + a_m + a_m^\dagger\right), \tag{18}$$

$$y \equiv i\sqrt{\frac{\hbar}{2m\tilde{\omega}_c}}\left(a_c - a_c^\dagger - a_m + a_m^\dagger\right). \tag{19}$$

Here we have defined the frequency $\tilde{\omega}_c \equiv \sqrt{\omega_c^2 - 2\omega_z^2}$, with $\omega_z \equiv (eV_0/mL^2)^{1/2}$ being the axial oscillation frequency. Its value depends on the voltage V_0 applied between the trap electrodes and on the characteristic trap size L.

After substituting equations (17), (18), and (19) into the interaction Hamiltonian, equation (16), we obtain

$$\begin{aligned} H_{drive} = g\mu_B b_1 &\sqrt{\frac{\hbar}{2m\tilde{\omega}_c}}\cos(\omega_d t + \phi) \\ &\times\left[\sigma_+(a_c + a_m^\dagger) + \sigma_-(a_c^\dagger + a_m)\right], \end{aligned} \tag{20}$$

which clearly shows how the electron spin, not only couples to the cyclotron oscillator, but also to the magnetron motion. However, with the help of the interaction picture

$$\sigma_\pm \longrightarrow \sigma_\pm \exp(\pm i\omega_s t), \tag{21}$$

$$a_c \longrightarrow a_c \exp(-i\omega_c t), \tag{22}$$

$$a_m \longrightarrow a_m \exp(+i\omega_m t), \tag{23}$$

one can see that this last effect is negligible when $\omega_d = \omega_a \equiv \omega_s - \omega_c$

$$H_{drive}^{(IP)} \simeq \frac{g}{2}\mu_B b_1\sqrt{\frac{\hbar}{2m\tilde{\omega}_c}}\left(\sigma_+ a_c e^{-i\phi} + \sigma_- a_c^\dagger e^{i\phi}\right). \tag{24}$$

The Hamiltonian, equation (24), has been derived in the rotating wave approximation, which allows to eliminate the fast rotating terms involving the magnetron motion.

Hence, the alternating magnetic field at the anomaly frequency provides the required interaction between the spin and the cyclotron qubits, that can be now manipulated with the composite pulse technique.

The corresponding unitary time evolution is given by

$$U(t) = \exp\left(-\frac{i}{\hbar}H_{drive}^{(IP)}t\right), \tag{25}$$

which, after some algebra, can be recasted as

$$U(t) = \mathcal{C}(t) + i\mathcal{S}(t), \tag{26}$$

where

$$\mathcal{C}(t) = \sigma_+\sigma_- \cos\left(\frac{\theta}{2}\sqrt{a_c a_c^\dagger}\right) + \sigma_-\sigma_+ \cos\left(\frac{\theta}{2}\sqrt{a_c^\dagger a_c}\right), \quad (27)$$

$$\mathcal{S}(t) = \sigma_+ e^{-i\phi} \frac{\sin\left(\frac{\theta}{2}\sqrt{a_c a_c^\dagger}\right)}{\sqrt{a_c a_c^\dagger}} a_c + \sigma_- e^{i\phi} \frac{\sin\left(\frac{\theta}{2}\sqrt{a_c^\dagger a_c}\right)}{\sqrt{a_c^\dagger a_c}} a_c^\dagger. \quad (28)$$

The parameter

$$\theta \equiv -g\mu_B b_1 \sqrt{\frac{1}{2m\hbar\tilde{\omega}_c}}\, t \quad (29)$$

depends on the strength and on the duration of the pulse.

Starting from the expression, equation (26), of the unitary evolution operator, we can construct its matrix representation in the subspace spanned by the vectors $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle, |02\rangle\}$

$$M(\theta, \phi) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & A & B & 0 & 0 \\ 0 & -B^* & A & 0 & 0 \\ 0 & 0 & 0 & C & D \\ 0 & 0 & 0 & -D^* & C \end{pmatrix}, \quad (30)$$

where

$$A \equiv \cos\left(\frac{\theta}{2}\right), \quad (31)$$

$$B \equiv ie^{i\phi} \sin\left(\frac{\theta}{2}\right), \quad (32)$$

$$C \equiv \cos\left(\frac{\theta}{\sqrt{2}}\right), \quad (33)$$

$$D \equiv ie^{-i\phi} \sin\left(\frac{\theta}{\sqrt{2}}\right). \quad (34)$$

A careful choice of the interaction time, such that $\theta = k\pi\sqrt{2}$ with k an integer, avoids populating the energy level $|02\rangle$ outside the computational subspace. We also note that the state $|00\rangle$ is left unchanged under the transformation equation (30), since it not coupled to any other energy level (see Fig. 1).

3.1 Swapping gate

The pulse sequence

$$M\left(\frac{\pi}{\sqrt{2}}, 0\right) M\left(\frac{2\pi}{\sqrt{2}}, \phi_S\right) M\left(\frac{\pi}{\sqrt{2}}, 0\right) \quad (35)$$

with $\phi_S \equiv \arccos[\cot^2(\pi/\sqrt{2})]$ generates the swapping gate between the cyclotron and the spin qubits. The first pulse $M(\pi/\sqrt{2}, 0)$ transfers the population from $|11\rangle$ to $|02\rangle$. Then the second pulse $M(2\pi/\sqrt{2}, \phi_S)$ performs the swapping operation between $|01\rangle$ and $|10\rangle$, without affecting any other state. Finally, the pulse $M(2\pi/\sqrt{2}, \phi_S)$ restores back the population from $|02\rangle$ to $|11\rangle$.

The swapping gate is essential to manipulate the cyclotron qubit. Indeed, we know how to perform any one-qubit rotation using the small oscillating magnetic field, equation (3), to control the spin state. Hence, we first exchange the information between the cyclotron and the spin qubits. Then we apply the desired one-qubit gate to the electron spin and, eventually, swap back the information to the cyclotron qubit. This final swapping operation is realized with the following pulses

$$M\left(\frac{\pi}{\sqrt{2}}, \pi\right) M\left(\frac{2\pi}{\sqrt{2}}, \pi + \phi_S\right) M\left(\frac{\pi}{\sqrt{2}}, \pi\right). \quad (36)$$

3.2 Conditional phase shift

The oscillatory magnetic field at the anomaly frequency, equation (24), is also useful to perform conditional dynamics on the cyclotron and spin qubits. For example, the two-qubit gate

$$|xy\rangle \longrightarrow e^{ixy\varphi}|xy\rangle \quad (37)$$

with $x, y = \{0, 1\}$, known as conditional phase shift, changes the phase of the quantum register if and only if both qubits are in the logic state 1. This operation is realized in our system with a sequence of four pulses

$$M\left(\frac{\pi}{\sqrt{2}}, \frac{\pi}{2}\right) M(\pi, 0)\, M\left(\frac{\pi}{\sqrt{2}}, \frac{\pi}{2}\right) M(\pi, 0). \quad (38)$$

Indeed, it is straightforward to prove that the most general state of a two-qubit register

$$\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle \quad (39)$$

is mapped, after the four pulses, onto

$$\alpha|00\rangle - \beta|01\rangle - \gamma|10\rangle - \delta|11\rangle. \quad (40)$$

Strictly speaking, apart from a global phase factor, the final state, equation (40), of the two-qubit register amounts to a change of π in the phase of $|00\rangle$ rather than of $|11\rangle$. A different encoding of the logical states into the physical states of the system solves this apparent inconsistency. The counterintuitive encoding, which assigns to the cyclotron state $|0\rangle$ ($|1\rangle$) and to the spin state $|\downarrow\rangle$ ($|\uparrow\rangle$) the logic value 1 (0), would lead to the expected result of equation (37), with $\varphi = \pi$.

4 Conclusions

The cyclotron oscillator and the spin of a trapped electron can be used to store and manipulate qubits of information. Universal logic gates are performed by means of external oscillatory magnetic fields. With the present proposal, we described a method to control the state of the cyclotron oscillator for quantum computation, without resorting to the anharmonicities. The trick is to send pulses of proper duration at the anomaly frequency, in order not to leave the computational subspace. This is accomplished with a small oscillating magnetic field coupling the cyclotron and the spin qubits. A similar set-up is already used in experiments aiming at the direct measurement of the electron anomaly [9]. We showed how to implement the swapping gate, which combined with the ability to prepare the electron spin in an arbitrary superposition, allows for realizing any single-qubit gate on both the cyclotron and the spin degrees of freedom. Moreover, the same mechanism can produce conditional dynamics. As an example, we provided the pulse sequence required to realize the conditional phase gate.

This work applies to a single trapped electron concepts and ideas proposed and developed in other contexts, like nuclear magnetic resonance [20] and ion traps. Quantum computation can only benefit from cross-field fertilization and other techniques could be exported as well between different systems. In this respect, the conference on "Quantum Information with Atoms, Ions and Photons", held in La Thuile, offered the best conditions to share ideas, compare results, and define new challenges in different areas of physics, in a joint effort towards a working quantum computer.

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